Optimal Controlled Variables for Parallel Process Units

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Innovation and Creativity

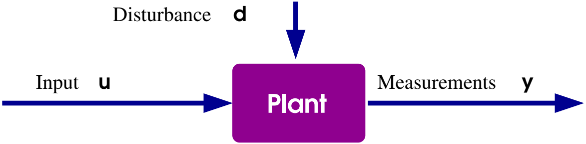
Main Idea & Outline of this talk

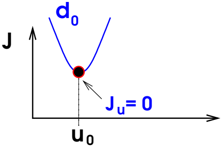
Main idea

Avoid online re-optimization when disturbances occur

Introduction

- Optimization of steady state operation

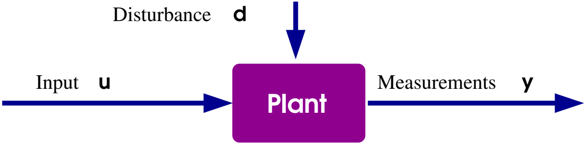


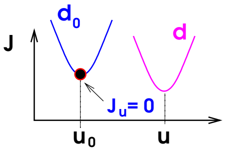


Goal: Simple implementation

Introduction

- Optimization of steady state operation

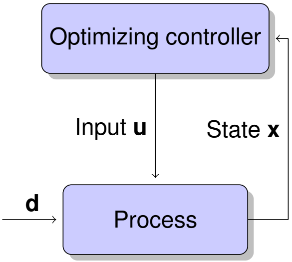




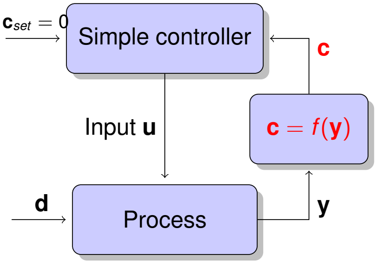
Goal: Simple implementation

Self-optimizing control

Traditional approach



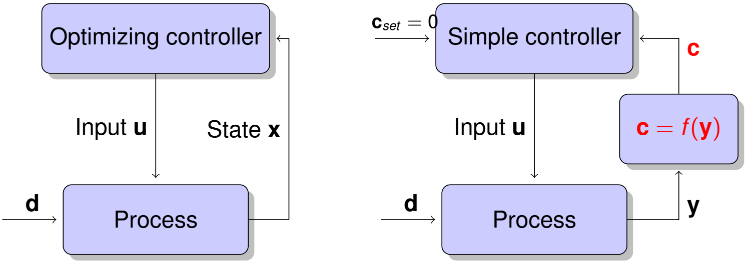
We propose:



Self-optimizing control

Traditional approach

We propose:



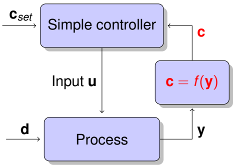
Skogestad (2000):

Self-optimizing control is if we can achieve an acceptable loss with constant setpoint values for the controlled variables

Self-optimizing control

Properties:

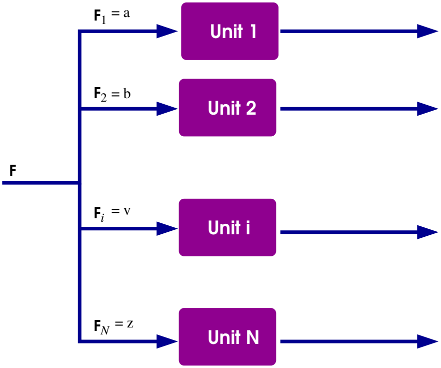
- Avoid (minimize) need for reoptimization
- Simple implementation
- Separating control and optimization
- Easy operator interaction
- In practice often: Little loss



In this talk:

A special application of this concept

Optimization of parallel processes



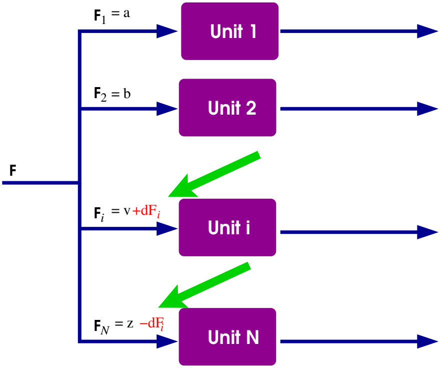
Optimization problem

$$\min_{F_1,\ldots,F_N} J = \sum_{i=1}^{N} J_i(F_i)$$

$$\text{s. t.} \quad F - \sum_{i=1}^{N} F_i = 0$$

- We have $N-1$ Degrees of freedom (DOF)
- Assume: Change dF_i is compensated by $dF_N = -dF_i$
- All other streams remain unchanged

Optimization of parallel processes



Optimization problem

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- We have $N - 1$ Degrees of freedom (DOF)
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- All other streams remain unchanged

Optimality Conditions

Unconstrained problem

$$\min_{F_1,\dots,F_{N-1}} \quad J = J_1(F_1) + \cdots + J_N(F - \sum_{i=1}^{N-1} F_i)$$

At optimum:

$$\frac{\partial J}{\partial F_i} = \frac{\partial J_i}{\partial F_i} + \frac{\partial J_N}{\partial F_i} = 0$$

Using $dF_i = -dF_N$:

Optimality condition

$$\frac{\partial J_i}{\partial F_i} - \frac{\partial J_N}{\partial F_N} = 0$$

N is chosen arbitrarily $\Longrightarrow$ holds for all pairs i, j

Principle of equal marginal utility (gain)

Well-known principle

- Equal marginal utility (costs):

$$\frac{\partial J}{\partial F_i} = \frac{\partial J}{\partial F_j}$$

- Decoupled, pairwise conditions!

Interpretation

At the optimum, the gain (utility) from increasing the load in one unit must be equal to the loss caused by decreasing the load in another unit.

Finding CVs for parallel processes

1. Optimality conditions:

$$\frac{\partial J}{\partial F_i} = \frac{\partial J_i}{\partial F_i} - \frac{\partial J_j}{\partial F_j}$$

2. Use a simple model to
   a.) express $\frac{\partial J_i}{\partial F_i}$
   b.) eliminate unknown variables (disturbances)

$$\gamma_i(\mathbf{y}_i) = \frac{\partial J_i}{\partial F_i}$$

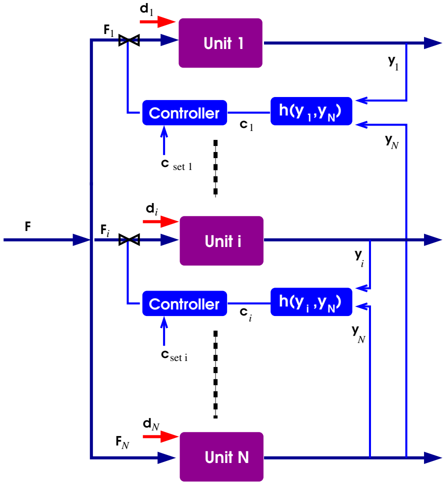
3. For all pairs i, j, control

$$c = h(\mathbf{y}_i, \mathbf{y}_j) = \gamma_i(\mathbf{y}_i) - \gamma_j(\mathbf{y}_j)$$

to zero

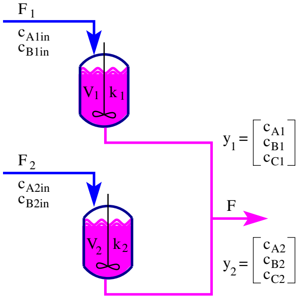
Self-optimizing control parallel processes

For each DOF we have a controlled variable $c = h(y_i - y_j)$



Keep c_i at constant setpoints, $c_{set} = 0$

General CSTR



- General Reaction

$$aA + bB \rightarrow cC + dD$$

- Reaction rate of "Y"

$$r_Y = \frac{1}{\nu_Y} k c_A^{\alpha} c_B^{\beta} V$$

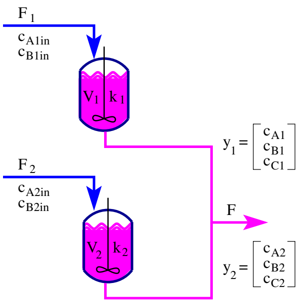
- Total flow given

$$F_1 + F_2 = F$$

Maximize revenue for product C

$$\max\ p_C F c_C = p_C F_1 c_{C1} + p_C F_2 c_{C2}$$

General CSTR



For each line $i = 1, 2$ we have

- Disturbances $\mathbf{d}_i$
  - $c_{Ai,in}$
  - $c_{Bi,in}$
  - k_i
  - V_i
- Measurements $\mathbf{y}_i$
  - c_{Ai}
  - c_{Bi}
  - c_{Ci}

Maximize revenue for product C

$$\max\ p_C F c_C = p_C F_1 c_{C1} + p_C F_2 c_{C2}$$

Optimality condition

Component balances

$$\begin{aligned} g_1 &= F_i c_{Ai,in} - F_i c_{Ai} - a k_i c_{Ai}^{\alpha} c_{Bi}^{\beta} V_i = 0 \\ g_2 &= F_i c_{Bi,in} - F_i c_{Bi} - b k_i c_{Ai}^{\alpha} c_{Bi}^{\beta} V_i = 0 \\ g_3 &= -F_i c_{Ci} + c k_i c_{Ai}^{\alpha} c_{Bi}^{\beta} V_i = 0 \\ g_3 &= -F_i c_{Di} + d k_i c_{Ai}^{\alpha} c_{Bi}^{\beta} V_i = 0. \end{aligned}$$

Marginal costs

$$\frac{\partial J_i}{\partial F_i} = p_C c k c_{Ai}^{\alpha} c_{Bi}^{\beta} V_i \frac{c_{Ai} c_{Bi}(\alpha + \beta) - \alpha c_{Ai,in} c_{Bi} - \beta c_{Ai} c_{Bi,in}}{c_{Ai} c_{Bi} F_i + \alpha a c_{Bi} k c_{Ai}^{\alpha} c_{Bi}^{\beta} V_i + \beta b c_{Ai} k c_{Ai}^{\alpha} c_{Bi}^{\beta} V_i}$$

- Cannot be used for control:
  - Contains unknowns k_i, V_i, $c_{Ai,in}$, $c_{Bi,in}$

Optimality conditions – Eliminating unknowns

Exact optimal controlled variable

$$c = \gamma_1 - \gamma_2$$

with

$$\gamma_i = c_{Ci} \frac{-p_C}{1 + \left(\frac{c c_{Ai} c_{Bi}}{c_{Ci}(\alpha a c_{Bi} + \beta b c_{Ai})} \right)}$$

Simple approximation ($c_{A_i}, c_{B_i} << c_{Ci}$)

$$\gamma_i \approx -p_c c_{Ci}$$

Approximate optimal controlled variable

$$c = c_{C1} - c_{C2}$$

- Gives optimal split for $aA + bB \rightarrow cC + dD$

Case Study

Goal

Given a total flow F adjust the loads F_i to maximize c_C

- Reaction:

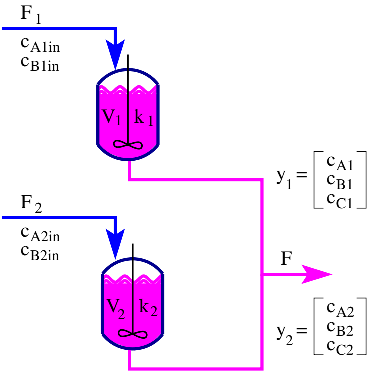
$$A + 2B \rightarrow 2C + D$$

- Reaction rate

$$r_A = -k c_A c_B^2$$

$$r_B = -\frac{1}{2} k c_A c_B^2$$

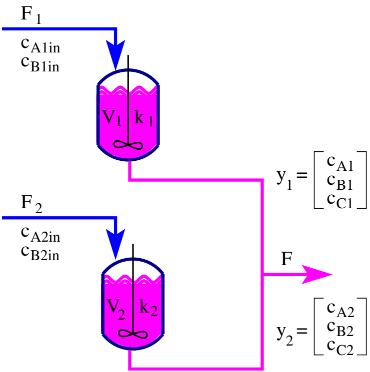
- Price $p_C = 1\$/mol$



Case Study – Scenario 1

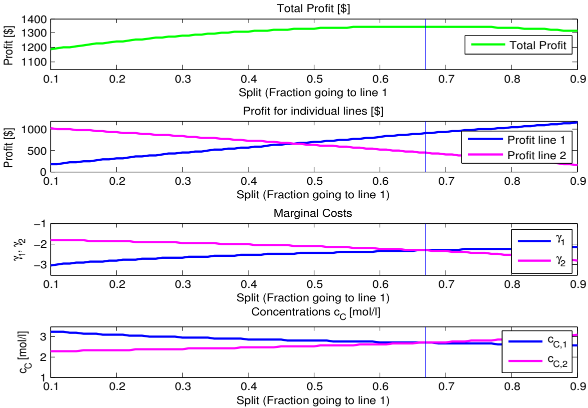
Disturbance

- Reaction constant k_2 decreases 50%



Case Study: Scenario 1

- Reaction constant k_1 drops 50%

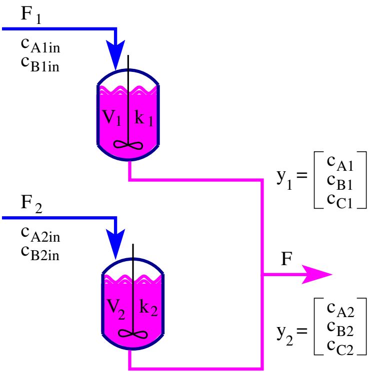


Savings: 120,000$/a

Case Study – Scenario 2

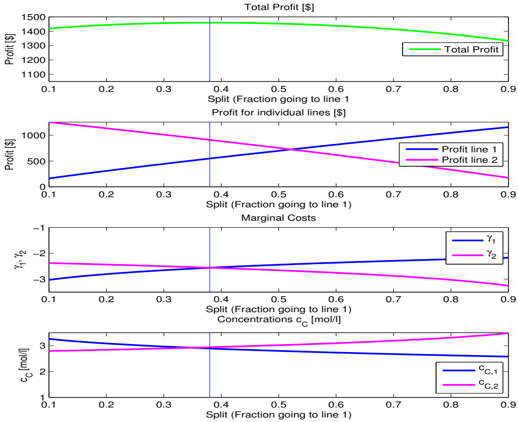
Disturbance

- $c_{B2,in}$ increases 10%



Case Study: Scenario 2

- Feed composition $c_{B,in}$ is increased 10%



Savings: 52,560$/a

Conclusion

- Very simple strategy:
  - Cheap
  - Pairwise controlled variables
    - Large problem decomposed to smaller subproblems
  - Robustness can be easily checked
  - Requires simple (approximate) models
    - Elimination possible
- CSTR Application
  - General reaction
  - Main "disturbance rejection capacity" in c_{ci}
  - Often sufficient: $c = c_{C1} - c_{C2}$
  - Side reactions can be included in the mass balances
- Suitable for practical implementation

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Thank you for your attention!